On Second Order Gauge Invariant Perturbations in Multi-Field Inflationary Models

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Abstract

In a recent letter [1] Acquaviva et. al presented results from a second order calculation for a single field inflationary model. In this paper we elaborate on their approach. We present equations for the second order superhorizon perturbations of a generic multi field model. We utilise a change of coordinates in field space - first presented in [2] and given a more geometrical flavour here - to separate isocurvature and adiabatic perturbations and construct gauge invariant variables related to them to second order. Explicit relations are given for two scalar fields on a flat field manifold although the results can be generalised to curved field manifolds and an arbitrary number of fields. This is an outline of a possible procedure to study nonlinear and nongaussian effects during multifield inflation. For a more detailed discussion we refer to a future publication [12].

1 Introduction

It has been customary to say that an adiabatic, gaussian, almost scale invariant perturbation was a generic prediction of inflation. In the past few years though it has been realised that if you allow for more than one degree of freedom to be relevant during inflation - i.e more dynamic scalar fields - then you can also have isocurvature perturbations, possibly correlated with the adiabatic ones, leading to a far richer phenomenology. For example simple statements about single field inflation such as the conservation of the superhorizon curvature perturbation $\mathcal{R}$ do not hold in multifield models [10]. Therefore the need for more accurate modeling of the inflationary era has arisen and is actively pursued at the moment.

So far almost all studies of inflationary perturbations have been performed using only linear perturbation theory. The smallness of the fluctuations in the temperature of the CMB certainly justify this approach. But given the accuracy of the forthcoming data it would be worth trying to go beyond this approximation and see if we can extract more information about our models by studying nonlinear effects during inflation. Non linearities would induce non gaussianities in the fluctuations of the cosmic microwave background which could be potentially observable by the PLANCK sattelite due for launch in 2007. The level of non-gaussianity in standard single field models of inflation has been estimated in the past (e.g [3, 15, 5]). It turns out that such a signal will not be detectable by PLANCK [4]. Multiple scalar fields seem to have a better chance of producing an observable nongaussian signal [6] the detection of which could provide evidence that more than one degree of freedom were relevant during inflation.

In this paper an attempt is made to formulate a method for the calculation, to the lowest order, of the nonlinear evolution of the perturbations generated from a generic multifield inflationary model. This is done by extending the usual perturbation theory to second order. A future paper will address the issue from a complementary perspective [12].

2 Perturbations and gauge invariance at second order

Cosmological perturbation theory is a rather arcane subject. The reason is that in a general perturbed spacetime there is no privileged coordinate system with respect to which one can define perturbations. So perturbations can change when we change the coordinates. Let us briefly recall what is usually meant when one talks of perturbations in general relativity [8]. One considers a five dimensional space composed of the background spacetime $\mathcal{M}_0$ and, stacked above it, perturbed spacetimes $\mathcal{M}_\epsilon$ parametrized by the parameter ϵ. We implicitly assume some sort of differentiable structure on this 5-D space such that these perturbed spacetimes can be considered "close" to $\mathcal{M}_0$. On these spacetimes live tensor fields T. On this 5-D space we impose a vector field $X = (1, X^\mu)$ the integral lines of which "pierce" the various $\mathcal{M}_\epsilon$s i.e X is not tangent to any of the $\mathcal{M}_\epsilon$. The integral curves of this vector field are used for identifying points on $\mathcal{M}_\epsilon$ with points on the background $\mathcal{M}_0$. The choice of X is completely arbitrary and is called a choice of *gauge*.

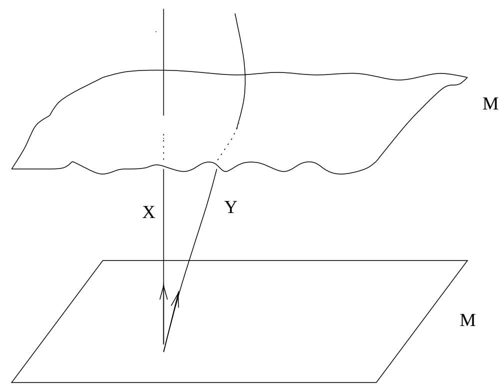

Fig. 1: Two vector fields X and Y live in a five dimensional space which is foliated by the background and the perturbed spacetimes. The perturbed spacetimes are parametrised by ϵ which caracterises the magnitude of the perturbations. Two different vector fields can be used to identify points on the perturbed spacetimes with points on the background and these different identifications correspond to different choices of gauge in conventional terminology. In this picture X is normal to the spacetimes $X = (1, 0)$.

In general any tensor T can be expanded as a taylor series

$$T_0 + \delta_X T = \phi^*_{X\epsilon}(T_\epsilon) = T_0 + \epsilon \pounds_X T\mid_0 + \frac{1}{2}\epsilon^2 \pounds_X \pounds_X T\mid_0 + \ldots, \quad (1)$$

or, calling the various terms the perturbations at various orders,

$$\phi^*_{X\epsilon}(T_\epsilon) = T_0 + \delta T^{(1)} + \delta T^{(2)} + \ldots, \quad (2)$$

where $\phi^*_{X\epsilon}$ is the pullback along X on the background manifold $\mathcal{M}_0$. Hence the vector field X alows us to identify points of the actual perturbed spacetimes with points of the background and hence define perturbations in a meaningful way.

Now consider another vector field $Y = (1, Y^\mu)$ different from X and use that for identifying points on the perturbed spacetimes with points on the background. This is a choice of a *different* gauge. Then, after a few manipulations of the lie derivatives, it can be shown that

$$\delta_Y T - \delta_X T = \epsilon \pounds_{\xi^{(1)}} T\mid_0 + \frac{1}{2}\epsilon^2 \pounds^2_{\xi^{(1)}} T\mid_0 + \epsilon^2 \pounds_{\xi^{(1)}} \delta^{(1)}_X T + \frac{1}{2}\epsilon^2 \pounds_{\xi^{(2)}} T\mid_0 + \ldots, \quad (3)$$

where $\delta_X^{(1)} T \equiv \epsilon \pounds_X T \mid_0$ is the linear perturbation of T in the "X gauge" and $\xi^{(1)} \equiv Y - X$, $\xi^{(2)} \equiv [X, Y]$ are vector fields which lie on $\mathcal{M}_0$ and are independent of each other. The first term in (3) is the usual result for the gauge transformation of the first order perturbations. The next three terms give the transformation for the second order ones. It is now easy to see that gauge fixing can be performed order by order. Once the gauge for the first order perturbations has been chosen, i.e a vector $\xi^{(1)}$ has been given, the vector field $\xi^{(2)}$ can be used to impose a gauge condition on the second order perturbations, independently of the first order gauge.

Expansion (3) also suggests a strategy for identifying gauge invariant quantities at second order. Observe that the first and fourth terms in (3) are essentially the same. This means that any linear combination $f(\delta T^{(1)})$ of first order variables which is gauge invariant to first order will also be gauge invariant w.r.t that part of the second order transformation which corresponds to the $\frac{1}{2}\epsilon^2 \pounds_{\xi^{(2)}} T \mid_0$ term in (3). The remaining terms $\frac{1}{2}\epsilon^2 \pounds^2_{\xi^{(1)}} T \mid_0 + \epsilon^2 \pounds_{\xi^{(1)}} \delta_X^{(1)} T$ are all composed of products of first order quantities. So in seeking gauge invariant combinations at second order we must look for appropriate quadratic terms of first order quantities that will cancel the corresponding quadratic terms in (3). If this can be done in a unique way then the form of a gauge invariant quantity at first order will dictate its form at second order.

We will now give an explicit example of the construction of a second order gauge invariant variable corresponding to the well known first order gauge invariant quantity [1]

$$\mathcal{R} = \psi + \frac{\mathcal{H}}{\varphi_0'} \delta\varphi. \tag{4}$$

In general every quantity will be expanded in orders like in (2). For example

$$\begin{aligned} g_{\mu\nu} &= g_{\mu\nu}^{(0)} + \delta g_{\mu\nu}^{(1)} + \frac{1}{2}\delta g_{\mu\nu}^{(2)} + ... \\ \varphi &= \varphi^{(0)} + \delta\varphi^{(1)} + \frac{1}{2}\delta\varphi^{(2)} + ... \end{aligned} \tag{5}$$

e.t.c. In particular, writing the general perturbed metric element as

$$ds^2 = a^2[(1+2\phi)d\tau^2 - 2B_i dx^i d\tau - [(1-2\psi)\gamma_{ij} + 2E_{ij}]dx^i dx^j] \tag{6}$$

we have

$$g_{00} = a(\tau)^2 \left(1 + 2\phi^{(1)} + \phi^{(2)} + ...\right) \tag{7}$$

$$g_{0i} = a(\tau)^2 \left(B_i^{(1)} + \frac{1}{2}B_i^{(2)} + ...\right) \tag{8}$$

$$g_{ij} = -a(\tau)^2 \left(1 - 2\psi^{(1)} - \psi^{(2)} + ... + 2E_{ij}^{(1)} + E_{ij}^{(2)} + ...\right) \tag{9}$$

Then from (3) one can calculate the formulae for the gauge transformations of the relevant quantities (see also [9]).

We would like to restrict our attention to super horizon modes and drop all second order spatial derivatives i.e take a long wavelength limit. We also impose the longitudinal gauge (which can be imposed at both first and second order) $\partial_i B^i = \partial_i E^i{}_j = 0$ where B_i is the shift and E_{ij} is the traceless part of the spatial perturbation defined earlier. Here we implicitly assume some sort of smoothing on superhorizon scales and we are dealing with smoothed fields. This means that we take the equations to act on quantities smoothed over superhorizon scales. For a general quantity Q we consider the equations to refer to the quantity

$$Q_L(x) \equiv \int d^3x' Q(x') W(|x - x'|/R) \tag{10}$$

where L stands for 'Long'. W is a window function rapidly decreasing for $|x - x'| > R$ and R is a smoothing scale larger than $\mathcal{H}^{-1}$. Of course this is not exact. Smoothing the full equations, i.e convolving the equations with the window function, is *not* equivalent to the original equations acting on smoothed fields in the case of nonlinear operators. For example, if Q satisfies

$$D(Q) = 0 \tag{11}$$

then

$$\int d^3x' D(Q(x'))W(|x - x'|/R) \neq D(Q_L) \tag{12}$$

if D is a nonlinear operator. Here we assume that $D(Q_L) = 0$ is an adequate description of the physics at long wavelengths. The rationale behind this approach is that while the system is essentially quantum mechanical it can be shown [13, 12] that when a mode becomes superhorizon it behaves like a classical yet random quantity in a well defined way. We assume that we can set initial conditions at horizon scales using linear perturbation theory - the quantization of which is well understood [14] - and then study their classical evolution on superhorizon scales using the classical nonlinear equations. Note that the perturbative approach which will be described later and is based on expansions such as expansion (1) or (7) - (9), doesn't seem to have any obvious meaning quantum mechanically. There is also another caveat. The smoothing will always introduce a cutoff and the dependence on this cutoff has to be dealt with. Such issues are left for future work ([12], see also [16]). We would like to note here that recently Maldacena [15] presented results regarding nonlinear quantum effects in an inflationary spacetime using the proper quantization procedure by computing qubic interaction terms in tne perturbed action (Standard Einstein-Hilbert action plus single scalar field), and then evaluating the three point correlators from them in a slow roll approximation. His method is a proper quantization with the interacting action which can yield an arbitrary N-point correlator (the calculation was done at tree level). Yet, knowing an arbitrary number of correlators doesn't seem to be very helpful in constructing actual nongaussian maps which will be useful for future explorations of the CMB (e.g the PLANCK satellite). We will return to this issue [12].

With the above things in mind we will ignore terms like $\partial_i f \partial_j g$ which will be of order $\mathcal{O}(\partial_i f \partial_j g) < \mathcal{H}^2$. We will also consider only scalar and tensor perturbations for simplicity since they seem to be the most important in most inflationary models. Then from (3) we have the following transformation rules for a gauge change at first order

$$\tilde{\phi}_{(1)} = \phi_{(1)} + {\xi^0_{(1)}}' + \mathcal{H}\xi^0_{(1)}, \tag{13}$$

$$\tilde{\psi}_{(1)} = \psi_{(1)} - \mathcal{H}\xi^0_{(1)}, \tag{14}$$

$$\tilde{\delta}\varphi_{(1)} = \delta\varphi_{(1)} + \varphi'\xi^0_{(1)}, \tag{15}$$

and at second order (*in the approximations mentioned*)

$$\begin{aligned}\tilde{\phi}_{(2)} = \phi_{(2)} + {\xi^0_{(2)}}' + \mathcal{H}\xi^0_{(2)} \quad &+ \quad \xi^0_{(1)}\left[2\left(\phi'_{(1)} + 2\mathcal{H}\phi_{(1)}\right) + {\xi^0_{(1)}}'' + 5\mathcal{H}{\xi^0_{(1)}}' + \left(\mathcal{H}' + 2\mathcal{H}^2\right)\xi^0_{(1)}\right] \\ &+ \quad 2{\xi^0_{(1)}}'\left(2\phi_{(1)} + {\xi^0_{(1)}}'\right),\end{aligned} \tag{16}$$

$$\tilde{\psi}_{(2)} = \psi_{(2)} - \mathcal{H}\xi^0 + \xi^0_{(1)}\left[2\left(\psi'_{(1)} + 2\mathcal{H}\psi_{(1)}\right) - \left(\mathcal{H}' + 2\mathcal{H}^2\right)\xi^0_{(1)} - \mathcal{H}{\xi^0_{(1)}}'\right], \tag{17}$$

$$\tilde{\delta}\varphi_{(2)} = \delta\varphi_{(2)} + \varphi'\xi^0_{(2)} + \xi^0_{(1)}\left(\varphi''\xi^0_{(1)} + \varphi'{\xi^0_{(1)}}' + 2\delta\varphi'_{(1)}\right). \tag{18}$$

Note that as mentioned before the part of the transformations containing the vector field $\xi_{(2)}$ is exactly the same as the first order case. From the above we see that the variable (4) at second order transforms like

$$\frac{\mathcal{H}}{\varphi_0'}\delta\tilde{\varphi}^{(2)} + \tilde{\psi}^{(2)} = \frac{\mathcal{H}}{\varphi_0'}\delta\varphi^{(2)} + \psi^{(2)} + \left[\mathcal{H}\frac{\varphi_0''}{\varphi_0'} - \left(\mathcal{H}' + 2\mathcal{H}^2\right)\right](\xi^0_{(1)})^2 + 2\left(\frac{\mathcal{H}}{\varphi_0'}\delta\varphi'_{(1)} + \psi'_{(1)} + 2\mathcal{H}\psi_{(1)}\right)\xi^0_{(1)} \quad (19)$$

As expected the transformation contains only products of first order quantities. Therefore we seek to construct a gauge invariant quantity at second order by adding a quadratic combination of first order quantities that will transform appropriately. By inspection we see that it must contain $\delta\varphi'$ and ψ' and it must not contain $\delta\varphi$. So we must have

$$\left(A\psi + C\psi' + D\delta\varphi'\right)\left(E\psi + G\psi' + H\delta\varphi'\right). \quad (20)$$

Noting that

$$\psi' \rightarrow \psi' - \mathcal{H}'\xi^0 - \mathcal{H}\xi^{0'} \quad (21)$$

$$\delta\varphi' \rightarrow \delta\varphi' + \varphi_0''\xi^0 + \varphi_0'\xi^{0'} \quad (22)$$

and that we must not have terms involving $\xi^{0'}$ we see that we have 2 options. We either set

$$D = C\frac{\mathcal{H}}{\varphi_0'} \quad (23)$$

$$H = G\frac{\mathcal{H}}{\varphi_0'} \quad (24)$$

which kills the terms involving $\xi^{0'}$ in the transformation or take

$$D = H = \frac{4\pi}{m_p^2}\mathcal{H}\varphi_0' \quad (25)$$

$$C = G = \mathcal{H}^2 - \mathcal{H}'. \quad (26)$$

and use the background equations of motion. In both cases we are forced to consider $A = E$, $C = G$ and $D = H$ and we end up with the same variable

$$\mathcal{R}_{(2)} = \left[\psi_{(2)} + \frac{\mathcal{H}}{\varphi_0'}\delta\varphi_{(2)}\right] + \frac{\left[\psi'_{(1)} + 2\mathcal{H}\psi_{(1)} + \frac{\mathcal{H}}{\varphi_0'}\delta_{(1)}\varphi'\right]^2}{\mathcal{H}' + 2\mathcal{H}^2 - \mathcal{H}\frac{\varphi_0''}{\varphi_0'}} \quad (27)$$

which is invariant under the transformations (17) and (18).

3 Einstein equations for a system of scalar fields at second order

Consider the perturbed line element

$$ds^2 = a^2[(1+2\phi)d\tau^2 - 2B_i dx^i d\tau - [(1-2\psi)\gamma_{ij} + 2E_{ij}]dx^i dx^j]. \quad (28)$$

Here it is understood that all quantities appearing are to be expanded as in (5). By inserting (28) into the Einstein equations with the relevant energy momentum tensor and keeping only linear order terms, one arrives at the well known equations of linear perturbation theory which can be symbolically represented as

$$\mathcal{D}\delta^{(1)}g = 0\,. \quad (29)$$

Here $\mathcal{D}$ is a set of linear differential operators and $\delta^{(1)}g$ represents the perturbation variables. At second order there are two types of terms. The $\delta^{(2)}g$'s and terms quadratic in the $\delta^{(1)}g$'s. The later are supposed to be known from the solution of the first order problem. The form of the equations are now

$$\mathcal{D}\delta^{(2)}g = \mathcal{J}[(\delta g)^2]\,. \tag{30}$$

with $\mathcal{D}$ the *same* operator as in (29) and $\mathcal{J}$ a "source" quadratic in the perturbations. Since the solution to the homogeneous equation $\mathcal{D}\delta g = 0$ is known then we can consider the source terms $\mathcal{J}[(\delta g)^2] \equiv \mathcal{J}[(\delta^{(1)}g)^2]$ as known functions to second order. The solution of (30) will have the form

$$\delta^{(2)}g(y) = \int \mathcal{D}^{-1}(y-x)\mathcal{J}\mathrm{d}x, \tag{31}$$

with $\mathcal{D}^{-1}(y-x)$ the appropriate Green's function, i.e the second order perturbations will be determined entirely by the $\mathcal{J}$'s. Here we implicitly assume that the appropriate initial conditions for the second order variables are homogeneous, i.e the nontrivial solution at second order is entirely due to the sources $\mathcal{J}$. If $\delta^{(1)}g$ is taken to be a gaussian random field, then (31) shows that $\delta^{(2)}g$ is given by the square of a gaussian random field ($\mathcal{J}$ is quadratic in first order perturbations) and is non-gaussian.

3.1 Gravity sector

In the longitudinal gauge, the linear perturbation of the Einstein tensor has the well known form

$$\begin{aligned}
\delta_L G^0{}_0 &= \frac{2}{a^2}\left[-3\mathcal{H}(\mathcal{H}\phi+\psi') + \nabla^2\psi\right], && (32)\\
\delta_L G^0{}_i &= \frac{2}{a^2}\partial_i\left(\mathcal{H}\phi+\psi'\right), && (33)\\
\delta_L G^i{}_j &= \frac{2}{a^2}\left([(2\mathcal{H}'+\mathcal{H}^2)\phi + \mathcal{H}\phi' + \psi'' + 2\mathcal{H}\psi' + \frac{1}{2}\nabla^2(\phi-\psi)]\right)\delta^i_j \\
&- \frac{1}{2}\partial_i\partial_j(\phi-\psi) + \frac{1}{a^2}\left[E''_{ij} + 2\mathcal{H}E'_{ij} - \nabla^2 E_{ij}\right], && (34)
\end{aligned}$$

where the subscript 'L' stands for the linear part. We also have for the energy momentum tensor

$$\begin{aligned}
\delta_L T^0{}_0 &= \frac{1}{a^2}[-\phi|\vec{\varphi}'_0|^2 + \vec{\varphi}'_0\cdot\vec{\delta\varphi}' + a^2\vec{\delta\varphi}\cdot V_{,\vec{\varphi}}], && (35)\\
\delta_L T^0{}_i &= \frac{1}{a^2}\vec{\phi}'_0\cdot\partial_i\vec{\delta\varphi}, && (36)\\
\delta_L T^i{}_j &= \frac{\delta^i{}_j}{a^2}[\phi|\vec{\varphi}'_0|^2 - \vec{\varphi}'_0\cdot\vec{\delta\varphi}' + a^2\vec{\delta\varphi}\cdot V_{,\vec{\varphi}}]. && (37)
\end{aligned}$$

Here we assume a multi scalar field model and $\vec{\varphi}_0$ and $\vec{\delta\varphi}$ denote the field vector and its perturbation. Given a basis $\hat{\mathbf{e}}_A$ in field space we have

$$\vec{\delta\varphi} = \delta\varphi^A\hat{\mathbf{e}}_A. \tag{38}$$

We use capital letters to denote coordinates in field space which we assume is flat. There is therefore a prefered basis, say $\hat{\mathbf{e}}_1, \hat{\mathbf{e}}_2$ which makes the kinetic term in the energy momentum tensor canonical. This is the one we use in (35) - (37). In general we could have a lagrangian of the form

$$\mathcal{L} = \frac{1}{2}G_{AB}\partial^\mu\varphi^A\partial_\mu\varphi^B - V(\varphi) \tag{39}$$

with $G_{AB} \neq \delta_{AB}$. Even in the flat case, a basis $\{\hat{\mathbf{e}}_A\}$ will in general depend on the coordinates in field space. Here, $\cdot$ denotes a product in field space. If we expand ϕ, ψ and $\delta\varphi^A$ as in (5) then δ_L is a linear operator acting at each order respectively.

At second order we will also have terms from the quadratic perturbation of the Einstein tensor and the energy momentum tensor which we denote by $\delta_2 G^\mu{}_\nu$ and $\delta_2 T^\mu{}_\nu$. Then at second order the Einstein equations become

$$\delta_L G^{(2)0}{}_0 = \frac{8\pi}{m_p^2}\delta_L T^{(2)0}{}_0 - \frac{2}{a^2}A \tag{40}$$

$$\delta_L G^{(2)0}{}_i = \frac{8\pi}{m_p^2}\delta_L T^{(2)0}i - \frac{2}{a^2}B_i \tag{41}$$

$$\delta_L G^{(2)i}{}_j = \frac{8\pi}{m_p^2}\delta_L T^{(2)i}j - \frac{2}{a^2}C^i{}_j \tag{42}$$

where

$$A \equiv \frac{a^2}{2}\left(\delta_2 G^0{}_0 - \frac{8\pi}{m_p^2}\delta_2 T^0{}_0\right) \tag{43}$$

$$B_i \equiv \frac{a^2}{2}\left(\delta_2 G^0{}_i - \frac{8\pi}{m_p^2}\delta_2 T^0{}_i\right) \tag{44}$$

$$C^i{}_j \equiv \frac{a^2}{2}\left(\delta_2 G^i{}_j - \frac{8\pi}{m_p^2}\delta_2 T^i{}_j\right). \tag{45}$$

(note that B_i here is different from the shift B_i of (28) - we are in the longitudinal gauge, and C has nothing to do with the C of the previous section). From (41), (33) and (36) we get that

$$(\mathcal{H}\phi + \psi') = \frac{4\pi}{m_p^2}\vec{\varphi}_0{}'\cdot\delta\vec{\varphi} - \nabla^{-2}(\partial_i B_i). \tag{46}$$

From (42), (34) and (37) we have

$$\delta_L G^{(2)i}{}_i = \frac{8\pi}{m_p^2}\delta_L T^{(2)i}{}_i - \frac{2}{a^2}C \tag{47}$$

or

$$\begin{aligned}(2\mathcal{H}' + \mathcal{H}^2)\phi + \mathcal{H}\phi' + \psi'' + 2\mathcal{H}\psi' + \frac{1}{3}\nabla^2(\phi - \psi) &= \frac{4\pi}{m_p^2}[-\phi|\vec{\varphi}_0'|^2 \\ &+ \vec{\varphi}_0'\cdot\delta\vec{\varphi}\,' - a^2\delta\vec{\varphi}\cdot V_{,\vec{\varphi}}] - \frac{1}{3}C.\end{aligned} \tag{48}$$

where $C = C^i{}_i$. Now use the background relation

$$|\vec{\varphi}_0'|^2 = \frac{m_p^2}{4\pi}(\mathcal{H}^2 - \mathcal{H}') \tag{49}$$

and (48) becomes

$$(\mathcal{H}\phi + \psi')' + 2\mathcal{H}\left(\mathcal{H}\phi + \psi'\right) + \frac{1}{3}\nabla^{-2}\left(\phi - \psi\right) = \frac{4\pi}{m_p^2}\left[\vec{\varphi}_0'\cdot\delta\vec{\varphi}\,' - a^2\delta\vec{\varphi}\cdot V_{,\vec{\varphi}}\right] - \frac{1}{3}C. \tag{50}$$

Using (46) and the background relation

$$\vec{\varphi}_0'' + 2\mathcal{H}\vec{\varphi}_0' + a^2 V_{\vec{\varphi}} = 0 \tag{51}$$

we get from (50) that

$$\frac{1}{3}\nabla^2\left(\phi-\psi\right)=-\frac{1}{3}C+\left(\nabla^{-2}\partial_i B_i\right)'+2\mathcal{H}\nabla^{-2}\partial_i B_i. \tag{52}$$

So we can write

$$\phi=\psi+K(\mathbf{x},\tau) \tag{53}$$

with

$$\frac{1}{3}\nabla^2 K(\mathbf{x},\tau)=-\frac{1}{3}C+\left(\nabla^{-2}\partial_i B_i\right)'+2\mathcal{H}\nabla^{-2}\partial_i B_i. \tag{54}$$

The r.h.s of (54) is a known 'source' term so $K(\mathbf{x},\tau)$ is in principle known. Combining the 00 and ij Einstein equations we get

$$\begin{aligned}\psi''+6\mathcal{H}\psi' &+ 2\left(\mathcal{H}'+2\mathcal{H}^2\right)\psi-\nabla^2\psi=-\frac{8\pi}{m_p^2}a^2V_{\vec{\varphi}}\cdot\delta\vec{\varphi}\\ &+ A-F'-2\mathcal{H}F-2(\mathcal{H}'+2\mathcal{H}^2)K-\mathcal{H}K'\end{aligned} \tag{55}$$

where

$$F\equiv\nabla^{-2}\partial_i B_i. \tag{56}$$

3.2 Matter sector

Equation (55) holds for an arbitrary number of fields. To express the $-\frac{8\pi}{m_p^2}a^2V_{\vec{\varphi}}\cdot\delta\vec{\varphi}$ term on the r.h.s it will be useful to use a basis in the scalar field space which is different from the canonical one. Such an idea was put forward in [2] in order to separate the entropy and adiabatic perturbations. Here we give a slightly more geometrical flavour which can be generalised to an arbitrary number of fields although we give explicit relations for two fields.

Given a background trajectory we can define a new set of basis forms through the relations

$$\mathbf{d}s=-\sin\theta\mathbf{d}\varphi^1+\cos\theta\mathbf{d}\varphi^2 \tag{57}$$

$$\mathbf{d}\sigma=\cos\theta\mathbf{d}\varphi^1+\sin\theta\mathbf{d}\varphi^2 \tag{58}$$

with

$$\cos\theta=\frac{\varphi^{1'}}{\sqrt{(\varphi^{1'})^2+(\varphi^{2'})^2}} \tag{59}$$

$$\sin\theta=\frac{\varphi^{2'}}{\sqrt{(\varphi^{1'})^2+(\varphi^{2'})^2}}. \tag{60}$$

Here, s and σ can be considered as two new coordinate functions. The function σ measures the length of the trajectory and defines a unit coordinate basis vector $\hat{\mathbf{e}}_\sigma=\frac{\partial}{\partial\sigma}$ tanjent to the trajectory. In particular

$$\hat{\mathbf{e}}_\sigma=\cos\theta\hat{\mathbf{e}}_1+\sin\theta\hat{\mathbf{e}}_2. \tag{61}$$

The form $\mathbf{d}s$ defines the dual vector $\hat{\mathbf{e}}_s=\frac{\partial}{\partial s}$ normal to the curve, i.e the coordinate s stays constant along the curve, with

$$\hat{\mathbf{e}}_s=-\sin\theta\hat{\mathbf{e}}_1+\cos\theta\hat{\mathbf{e}}_2 \tag{62}$$

From these we can get that

$$\hat{\mathbf{e}}_s'=-\theta'\hat{\mathbf{e}}_\sigma \tag{63}$$

$$\hat{\mathbf{e}}_\sigma'=\theta'\hat{\mathbf{e}}_s. \tag{64}$$

Note also that

$$\hat{\mathbf{e}}_\sigma = \frac{\vec{\varphi}'}{|\vec{\varphi}'|} \Rightarrow \vec{\varphi}'' = |\vec{\varphi}'|'\hat{\mathbf{e}}_\sigma + \theta'\hat{\mathbf{e}}_s|\vec{\varphi}'|. \tag{65}$$

Then we can use the background equation of motion

$$\vec{\varphi}'' + 2\mathcal{H}\vec{\varphi}' + a^2 V_{\vec{\varphi}} = 0 \tag{66}$$

to see that

$$|\vec{\varphi}'|' + 2\mathcal{H}|\vec{\varphi}'| + a^2 V_\sigma = 0 \tag{67}$$

and that

$$\theta' = -a^2 \frac{V_s}{|\vec{\varphi}'|} \tag{68}$$

Now, since $|d\vec{\varphi}| = d\sigma$, we see that (67) is

$$\sigma'' + 2\mathcal{H}\sigma' + a^2 V_\sigma = 0 \tag{69}$$

and

$$\theta' = -a^2 \frac{V_s}{\sigma'}. \tag{70}$$

We now want to reexpress the $-\frac{8\pi}{m_p^2} a^2 V_{\vec{\varphi}} \cdot \delta\vec{\varphi}$ term on the r.h.s of eqn. (55). We have that

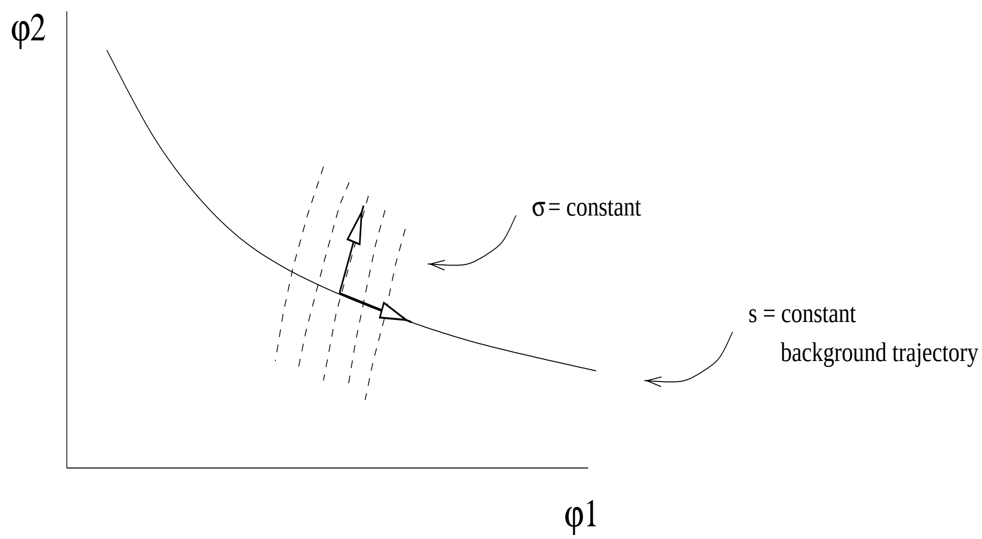

Fig. 2: The change of coordinates in field space. σ is the length of the background trajectory and s is constant along it. The two vectors are the basis vectors associated with these coordinates.

$$-\frac{8\pi}{m_p^2} a^2 V_{\vec{\varphi}} \cdot \delta\vec{\varphi} = \frac{8\pi}{m_p^2} (\vec{\varphi}'' + 2\mathcal{H}\vec{\varphi}') \cdot \delta\vec{\varphi}. \tag{71}$$

From (46)

$$\frac{4\pi}{m_p^2} \vec{\varphi}' \cdot \delta\vec{\varphi} = \frac{4\pi}{m_p^2} \sigma' \hat{\mathbf{e}}_\sigma \cdot \delta\vec{\varphi} \equiv \frac{4\pi}{m_p^2} \sigma' \delta\sigma = (\mathcal{H}\psi + \psi') + F + \mathcal{H}K \tag{72}$$

where we have defined $\delta\sigma$ to be the part of the perturbation along the trajectory, K is defined in (53) and (54) and F in (56). Using

$$\vec{\varphi}'' = \sigma'' \hat{\mathbf{e}}_\sigma + \sigma' \theta' \hat{\mathbf{e}}_s \tag{73}$$

and (72) we can get from (55) the following equation for the gravitational potential

$$\psi'' + 2\left(\mathcal{H} - \frac{\sigma''}{\sigma'}\right)\psi' \quad + \quad 2\left(\mathcal{H}' - \mathcal{H}\frac{\sigma''}{\sigma'}\right)\psi = \frac{8\pi}{m_p^2}\left(\vec{\varphi}_0'' \cdot \hat{\mathbf{e}}_s\right)\delta s$$
$$\quad + \quad A - F' + 2\frac{\sigma''}{\sigma'}F + 2\left(\frac{\sigma''}{\sigma'}\mathcal{H} - \mathcal{H}'\right)K - \mathcal{H}K' \qquad (74)$$

where for the particular case of two fields $\vec{\varphi}_0'' \cdot \hat{\mathbf{e}}_s = \sigma'\theta'$ and we have dropped the $\nabla^2\psi$ term. We see that the perpendicular perturbation δs acts as a source for the gravitational potential [2] and at second order we have extra terms on the r.h.s of (74). Since we now have more degrees of freedom than the single field case we need an extra equation for δs. This we can get from the scalar equation of motion

$$\nabla^\mu \nabla_\mu \left(\varphi^A \hat{\mathbf{e}}_A\right) + G^{AB}\hat{\mathbf{e}}_A \frac{\partial V}{\partial \varphi^B} = 0. \qquad (75)$$

Here G^{AB} is the (inverse) metric associated with the new basis, i.e

$$G_{AB} = \hat{\mathbf{e}}_A \cdot \hat{\mathbf{e}}_B, \;\; G_{AB}G^{BC} = \delta^A_C. \qquad (76)$$

An arbitrary basis $\{\hat{\mathbf{e}}_A\}$ will in general depend on the field coordinates. Such is the case for our new basis which therefore depends implicitly on time as we move along the trajectory. The spacetime covariant derivative of the basis vectors will be

$$\nabla_\mu \hat{\mathbf{e}}_A = \partial_\mu \varphi^B \mathcal{D}_B \hat{\mathbf{e}}_A \equiv \partial_\mu \varphi^B \Gamma^C{}_{AB}\hat{\mathbf{e}}_C \qquad (77)$$

where $\mathcal{D}_B$ is the covariant derivative in field space and $\Gamma^A{}_{BC}$ are the corresponding connection coefficients. We now need to compute G_{AB} and its derivatives in order to compute the connection. By consrtuction $G_{AB} = \delta_{AB}$ on the background trajectory. To calculate its derivatives we note the following. The functions s and σ are new coordinate functions and the basis we constructed is a coordinate basis which implies

$$\hat{\mathbf{e}}_s = \partial/\partial s\,, \qquad (78)$$
$$\hat{\mathbf{e}}_\sigma = \partial/\partial\sigma = \frac{1}{\sigma'}\frac{d}{d\tau}\,, \qquad (79)$$
$$[\hat{\mathbf{e}}_s, \hat{\mathbf{e}}_\sigma] = 0. \qquad (80)$$

By acting with the commutator (80) on the original coordinate functions i.e. the fields φ^1 and φ^2 we get that

$$\frac{\partial}{\partial s}\cos\theta = -\frac{\theta'}{\sigma'}\cos\theta\,, \qquad (81)$$
$$\frac{\partial}{\partial s}\sin\theta = -\frac{\theta'}{\sigma'}\sin\theta. \qquad (82)$$

With these and (79) we can calculate

$$\frac{\partial \hat{\mathbf{e}}_s}{\partial s} = -\frac{\theta'}{\sigma'}\hat{\mathbf{e}}_s, \qquad (83)$$
$$\frac{\partial \hat{\mathbf{e}}_\sigma}{\partial s} = -\frac{\theta'}{\sigma'}\hat{\mathbf{e}}_\sigma, \qquad (84)$$
$$\frac{\partial \hat{\mathbf{e}}_s}{\partial \sigma} = -\frac{\theta'}{\sigma'}\hat{\mathbf{e}}_\sigma, \qquad (85)$$
$$\frac{\partial \hat{\mathbf{e}}_\sigma}{\partial \sigma} = +\frac{\theta'}{\sigma'}\hat{\mathbf{e}}_s. \qquad (86)$$

Now

$$\frac{\partial}{\partial\varphi^C}G_{AB} = \frac{\partial\hat{\mathbf{e}}_A}{\partial\varphi^C}\cdot\hat{\mathbf{e}}_B + \frac{\partial\hat{\mathbf{e}}_B}{\partial\varphi^C}\cdot\hat{\mathbf{e}}_A. \tag{87}$$

The only non zero derivatives are

$$G_{ss,s} = G_{\sigma\sigma,s} = -2\frac{\theta'}{\sigma'} \tag{88}$$

which give the connection coefficients

$$\Gamma^s{}_{\sigma\sigma} = -\Gamma^\sigma{}_{\sigma s} = -\Gamma^s{}_{ss} = \frac{\theta'}{\sigma'} \tag{89}$$

all the rest being zero. Note that these results are compatible with eqn's (63) and (64).

Now equation (75) upon expansion to second order gives

$$\nabla^\mu\nabla_\mu\left(\delta\varphi^A\hat{\mathbf{e}}_A\right) + \delta\varphi^C\mathcal{D}_C\left(G^{AB}\hat{\mathbf{e}}_A\frac{\partial V}{\partial\varphi^B}\right) = -J^A\hat{\mathbf{e}}_A \tag{90}$$

where $J^A\hat{\mathbf{e}}_A$ is the quadratic part of the perturbed equation (see Appendix). Now we need to project out the part normal to the trajectory of the fields by contracting with $\mathbf{d}s$. Using the results found above, we find that the equation for the perpendicular field perturbation δs at second order is

$$\begin{aligned}\delta s'' + 2\mathcal{H}\delta s' + \left(a^2V_{ss} - \theta'^2\right)\delta s &= 2a^2\theta'\sigma'\psi - 2a^2\theta'\delta\sigma' \\ &+ 2a^2\left(\sigma'' - \mathcal{H}\sigma'\right)\frac{\theta'}{\sigma'}\delta\sigma - a^{-2}\langle J^A\hat{\mathbf{e}}_A\,,\,\mathbf{d}s\rangle \end{aligned}\tag{91}$$

where by $a^{-2}\langle J^A\hat{\mathbf{e}}_A, \mathbf{d}s\rangle$ we denote the contraction of the vector $J^A\hat{\mathbf{e}}_A$ with the basis form $\mathbf{d}s$, i.e its s component. We can eliminate $\delta\sigma$ from the above equation using the 00 and $0i$ Einstein equations to finaly get at second order

$$\delta s'' + 2\mathcal{H}\delta s' + \left(a^2V_{ss} + 3\theta'^2\right)\delta s = -a^{-2}\langle J^A\hat{\mathbf{e}}_A\,,\,\mathbf{d}s\rangle - 2a^2\frac{\theta'}{\sigma'}\frac{m_p^2}{4\pi}3\mathcal{H}F. \tag{92}$$

Equations (74) and (92) form a complete set to study the perturbations of a two field inflationary model. They have the same form as the linear ones except for the source terms appearing on the r.h.s. These sources are quadratic functions of first order perturbations and can be considered known. Expressions for them are given in the Appendix. Note that at second order the first order adiabatic perturbations source the second order isocurvature ones - the term J^A contains terms which depend on the first order adiabatic perturbation $\delta\sigma$, and the first order isocurvature source the second order gravitational potential - see eqn. (74), the sources on the r.h.s depend on the first order isocurvature perturbations - even if $\theta' = 0$. This is contrary to what happens in the linear case [2]. Therefore nonlinear effects seem to mix these two in a more complicated way than in the linear case.

We can now construct gauge invariant quantities in terms of the new field coordinates σ and s. At first order the gauge invariant curvature perurbation $\mathcal{R}$ is can be written as

$$\mathcal{R}_{(1)} = \left[\psi_{(1)} + \frac{\mathcal{H}}{\sigma'}\delta\sigma_{(1)}\right] \tag{93}$$

Since $s' = 0$, δs is gauge invariant at first order. At second order, according to eqn. (15)

$$\tilde{\delta}s = \delta s + 2\xi^0_{(1)}\delta s'_{(1)} \tag{94}$$

so a corresponding gauge invariant quantity is easily seen to be

$$\delta s^{(g.i)} = \delta s - 2\frac{\delta\sigma_{(1)}}{\sigma'}\delta s'_{(1)}. \tag{95}$$

Similarly we can construct a gauge invariant curvature perturbation at second order similar to (23)

$$\begin{aligned} \mathcal{R}_{(2)} &= \left[\psi_{(2)} + \frac{\mathcal{H}}{\sigma'}\delta\sigma_{(2)}\right] \\ &+ \frac{\left[\psi'_{(1)} + 2\mathcal{H}\psi_{(1)} + \frac{\mathcal{H}}{\sigma'}\delta_{(1)}\sigma'\right]^2}{\mathcal{H}' + 2\mathcal{H}^2 - \mathcal{H}\frac{\sigma''}{\sigma'}}. \end{aligned} \tag{96}$$

The variables (95) and (96) are second order gauge invariant variables (at least on superhorizon scales) which can be used to study isocurvature and adiabatic perurbations in the mild nonlinear regime.

4 Summary

We have touched upon the issue of gauge invariant variables for second order perturbations in multifield inflationary models and constructed such variables - equations (95) and (96) - on supehorizon scales. We have given a more geometrical flavour to the splitting of isocurvature and adiabatic perturbations and although explicit relations were given for the two field flat case, the results are extendible to multifield nonflat models. We also described a perturbative approach by which one can study the nonlinear evolution of perturbations in such models to second order in a perturbative expansion. The resulting equations - eqns. (74) and (92) - have the same form as the first order ones but with new terms appearing on the r.h.s. These new terms are quadratic in the first order perturbations and therefore they can be considered as known "sources". Such a formalism could be used to calculate the amount of nongaussianity produced in such models [12]. There are a few caveats however. We have implicitly assumed a smoothing on scales larger than the horizon. This will introduce a cutoff when we take the Fourier transform of the r.h.s sources in (74) and (92) and the dependence on this cutoff remains to be investigated. Also the resulting source terms, although in principle known, are quite complicated. We will return to the issue with a different approach in a future publication [12].

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A Appendix

In this appendix we give formulae nessecary for the calculation of the terms A, C, F, J^A defined in the text. These are the terms containing products of first order perturbations.

We write the metric as

$$g_{\mu\nu} = g^B_{\mu\nu} + h_{\mu\nu}. \tag{97}$$

where $h_{0i} = 0$. Indices will be raised and lowered with the background metric. Then the perturbation of the contravariant metric tensor will be

$$g^{\mu\nu} = g^{(B)\mu\nu} - h^{\mu\nu} + h^{\mu}{}_{\alpha}h^{\alpha\nu} \tag{98}$$

and in particular

$$g^{00} = \frac{1}{a^2} - \frac{1}{a^4}h_{00} + \frac{1}{a^6}h_{00}h_{00} \tag{99}$$

$$g^{0i} = 0 \quad (100)$$

$$g^{ij} = -\frac{\delta_{ij}}{a^2} - \frac{1}{a^4}h_{ij} - \frac{1}{a^6}h_{ik}h_{kj}. \quad (101)$$

The perturbation of the metric determinant is

$$\sqrt{-g} = a^4\left(1 + \frac{1}{2}h + \frac{1}{8}h^2 - \frac{1}{4}h^{\alpha}{}_{\beta}h^{\beta}{}_{\alpha}\right). \quad (102)$$

The perturbation of the Riemann tensor to second order can be found to be (see also [11])

$$\begin{aligned} R^{(2)}_{\mu\nu} &= \frac{1}{2}\Bigg[\frac{1}{2}h_{\alpha\beta|\mu}h^{\alpha\beta}{}_{|\nu} + h^{\alpha\beta}\left(h_{\alpha\beta|\mu\nu} + h_{\mu\nu|\alpha\beta} - h_{\alpha\mu|\nu\beta} - h_{\alpha\nu|\mu\beta}\right) \\ &+ h_{\nu}{}^{\alpha|\beta}\left(h_{\alpha\mu|\beta} - h_{\beta\mu|\alpha}\right) - \left(h^{\alpha\beta}{}_{|\beta} - \frac{1}{2}h^{|\alpha}\right)\left(h_{\alpha\mu|\nu} + h_{\alpha\nu|\mu} - h_{\mu\nu|\alpha}\right)\Bigg]. \end{aligned} \quad (103)$$

where a vertical bar denotes a covariant derivative w.r.t. the background. Then

$$R^{(2)\mu}{}_{\nu} = g^{(B)\mu\alpha}R^{(2)}_{\alpha\nu} + \delta_{(1)}g^{\mu\alpha}R_{(1)\alpha\nu} + \delta_{(2)}g^{\mu\alpha}R^{(B)}_{\alpha\nu}. \quad (104)$$

Using the following perturbed metric tensor (longitudinal gauge)

$$h_{00} = 2a^2\psi \quad (105)$$

$$h_{ij} = 2a^2\psi\delta_{ij} - 2a^2E_{ij} \quad (106)$$

$$h = -4\psi \quad (107)$$

we get, excluding second order spatial derivatives

$$R^{(2)0}{}_{0} = \frac{1}{2a^2}\left[-24\mathcal{H}\psi'\psi + 2E'_{ij}E'_{ij} + 4E_{ij}\left(E''_{ij} + \mathcal{H}E'_{ij}\right)\right] - \frac{12}{a^2}\psi^2\mathcal{H}' \quad (108)$$

$$\begin{aligned} R^{(2)i}{}_{j} &= \frac{1}{2a^2}[-8\mathcal{H}\psi\psi'\delta_{ij} - 4(\psi')^2\delta_{ij} - 8\mathcal{H}\psi'E_{ij} - 4E_{ik}E''_{kj} + 4\mathcal{H}E_{il}{}'E_{jl} \\ &+ 4\mathcal{H}E'_{kl}E_{kl}\delta_{ij} + 8\mathcal{H}E_{ik}E'_{kj} - 4E_{ij}\psi''] \\ &- \frac{1}{a^2}\left(2\mathcal{H}^2 + \mathcal{H}'\right)\left(4\psi^2\delta_{ij} - 2\psi E_{ij} + E_{il}E_{lj}\right). \end{aligned} \quad (109)$$

Therefore the Einstein tensor will be

$$G^{(2)0}{}_{0} = \frac{1}{2}\left(R^{(2)0}{}_{0} - R^{(2)l}{}_{l}\right) \quad (110)$$

$$G^{(2)i}{}_{j} = R^{(2)i}{}_{j} - \frac{1}{2}\delta^{i}{}_{j}\left(R^{(2)0}{}_{0} - R^{(2)i}{}_{i}\right) \quad (111)$$

so we find

$$\begin{aligned} G^{(2)0}{}_{0} &= \frac{1}{a^2}\Bigg[3\psi'^2 + 12\mathcal{H}^2\psi^2 - 3\mathcal{H}E'_{kl}E_{kl} + \frac{8}{3}E''_{kl}E_{kl} + \frac{2}{3}E'_{kl}E'_{kl} \\ &+ \frac{2}{3}\left(2\mathcal{H}^2 + \mathcal{H}'\right)E_{kl}E_{kl}\Bigg] \end{aligned} \quad (112)$$

$$\begin{aligned} \frac{1}{3}G^{(2)i}{}_{i} &= \frac{1}{a^2}\Bigg[-8\mathcal{H}\psi'\psi + \psi'^2 + 4(\mathcal{H}^2 + 2\mathcal{H}')\psi^2 - \frac{1}{2}E'_{kl}E'_{kl} \\ &- \frac{2}{3}E''_{kl}E_{kl} - \frac{7}{3}\mathcal{H}E'_{kl}E_{kl} + \frac{1}{6}\left(2\mathcal{H}^2 + \mathcal{H}'\right)E_{kl}E_{kl}\Bigg]. \end{aligned} \quad (113)$$

For the energy momentum tensor

$$T^\mu_\nu = \partial^\mu(\vec{\varphi}+\vec{\delta\varphi})\cdot\partial_\nu(\vec{\varphi}+\vec{\delta\varphi}) - \delta^\mu_\nu\left[\frac{1}{2}\partial^\lambda(\vec{\varphi}+\vec{\delta\varphi})\cdot\partial_\lambda(\vec{\varphi}+\vec{\delta\varphi}) - V\right] \tag{114}$$

the second order perturbation is given by

$$\begin{aligned} T^{(2)0}{}_0 &= \frac{2}{a^2}\psi^2\sigma'^2 - \frac{2}{a^2}\psi\left(\sigma'\delta\sigma' - \sigma'\theta'\delta s\right) + \frac{1}{2a^2}\left(\delta s' + \theta'\delta\sigma\right)^2 \\ &+ \frac{1}{2a^2}\left(\delta\sigma' - \theta'\delta s\right)^2 + \frac{1}{2}V_{AB}\delta\varphi^A\delta\varphi^B + \delta\varphi^A\delta\varphi^B\Gamma^C{}_{AB}V_C \end{aligned} \tag{115}$$

$$\begin{aligned} T^{(2)i}{}_j &= -\delta^i{}_j\Big[\frac{2}{a^2}\psi^2\sigma'^2 - \frac{2}{a^2}\psi\left(\sigma'\delta\sigma' - \sigma'\theta'\delta s\right) + \frac{1}{2a^2}\left(\delta s' + \theta'\delta\sigma\right)^2 \\ &+ \frac{1}{2a^2}\left(\delta\sigma' - \theta'\delta s\right)^2 - \frac{1}{2}V_{AB}\delta\varphi^A\delta\varphi^B - \delta\varphi^A\delta\varphi^B\Gamma^C{}_{AB}V_C\Big] \end{aligned} \tag{116}$$

where we have obviously dropped second order spatial derivatives and the connection coefficients $\Gamma^A{}_{BC}$ have been calculated in the text. We also need to compute the quadratic part of the wave equation for the system of scalar fields

$$\begin{aligned} \nabla^\mu\nabla_\mu\left(\varphi^A\hat{\mathbf{e}}_A + \delta\varphi^A\hat{\mathbf{e}}_A\right) &+ G^{AB}\frac{\partial V}{\partial\varphi^B}\hat{\mathbf{e}}_A + \delta\varphi^C G^{AB}\mathcal{D}_C\left(\frac{\partial V}{\partial\varphi^B}\hat{\mathbf{e}}_A\right) \\ &+ \frac{1}{2}\delta\varphi^C\delta\varphi^F G^{AB}\mathcal{D}_F\mathcal{D}_C\left(\frac{\partial V}{\partial\varphi^B}\hat{\mathbf{e}}_A\right) = 0. \end{aligned} \tag{117}$$

Here $\mathcal{D}$ is the covariant derivative in the field space. Up to second order spatial derivatives we have

$$\begin{aligned} J^A\hat{\mathbf{e}}_A = &- \left[a^2E_{kl}E_{kl}\hat{\mathbf{e}}_\sigma\right]' - \left[4a^2\psi(\delta s' + \theta'\delta\sigma)\hat{\mathbf{e}}_s\right]' - \left[4a^2\psi(\delta\sigma' - \theta'\delta s)\hat{\mathbf{e}}_\sigma\right]' \\ &+ (\delta_2\sqrt{-g})G^{AB}\hat{\mathbf{e}}_B V_A + \frac{1}{2}\sqrt{-g}G^{AB}\delta\varphi^C\delta\varphi^F\mathcal{D}_C\mathcal{D}_F(\hat{\mathbf{e}}_B V_A) \\ &+ (\delta_L\sqrt{-g})G^{AB}\delta\varphi^C\mathcal{D}_C(\hat{\mathbf{e}}_B V_A) \end{aligned} \tag{118}$$

where

$$\delta_2\sqrt{-g} = -a^4(2\psi^2 + E_{kl}E_{kl}) \tag{119}$$

$$\delta_L\sqrt{-g} = -2a^4\psi \tag{120}$$

With these formulae we can compute all the second order terms defined in the text.

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